7.1   Transmission of Prestress (Part I)

This section covers the following topics.

- Pre-tensioned Members

7.1.1 Pre-tensioned Members

The stretched tendons transfer the prestress to the concrete leading to a self equilibrating system. The mechanism of the transfer of prestress is different in the pre-tensioned and post-tensioned members. The transfer or transmission of prestress is explained for the two types of members separately.

For a pre-tensioned member, usually there is no anchorage device at the ends. The following photo shows that there is no anchorage device at the ends of the pre-tensioned railway sleepers.

**Figure 7-1.1**   End of pre-tensioned railway sleepers

(Courtesy: The Concrete Products and Construction Company, COPCO, Chennai)

For a pre-tensioned member without any anchorage at the ends, the prestress is transferred by the bond between the concrete and the tendons. There are three mechanisms in the bond.

1) Adhesion between concrete and steel
2) Mechanical bond at the concrete and steel interface

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3) Friction in presence of transverse compression.

The mechanical bond is the primary mechanism in the bond for indented wires, twisted strands and deformed bars. The surface deformation enhances the bond. Each of the type is illustrated below.
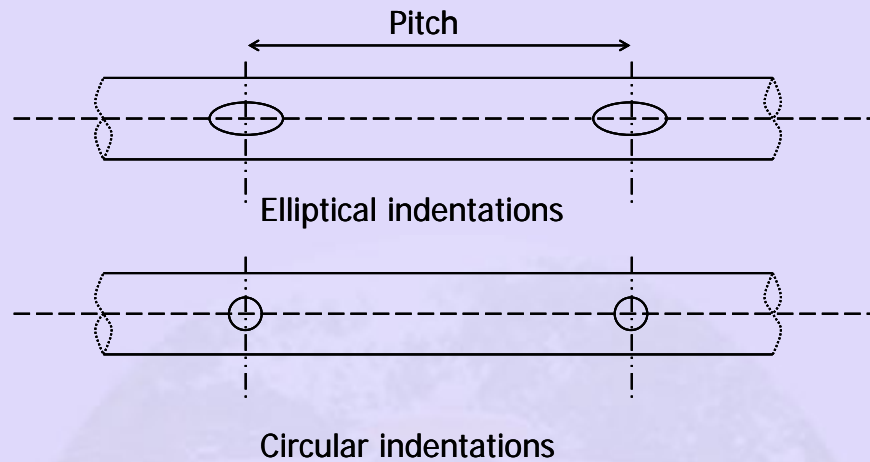
Examples of indented wires
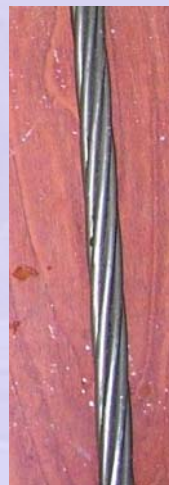
Figure 7-1.2   Indented wires, twisted strands and deformed bars

The prestress is transferred over a certain length from each end of a member which is called the **transmission length** or **transfer length** (L_t). The stress in the tendon is zero at the ends of the members. It increases over the transmission length to the effective prestress (f_{pe}) under service loads and remains practically constant beyond it.   The following figure shows the variation of prestress in the tendon.
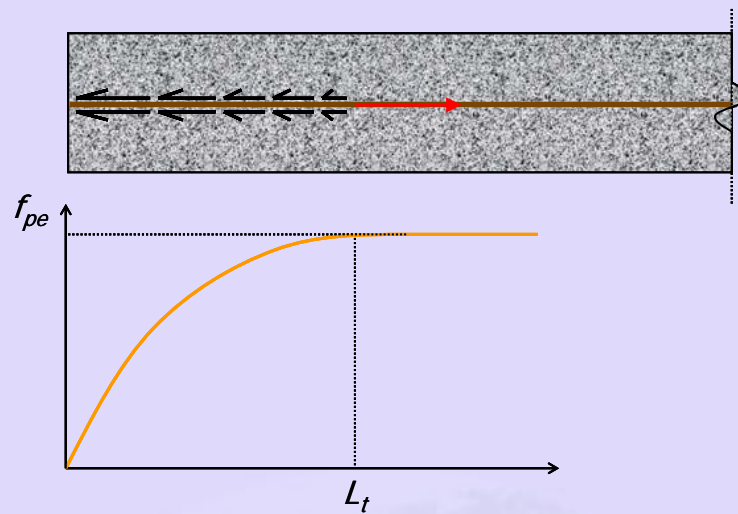
Figure 7-1.3   Variation of prestress in tendon along transmission length

Hoyer Effect

After stretching the tendon, the diameter reduces from the original value due to the Poisson's effect. When the prestress is transferred after the hardening of concrete, the ends of the tendon sink in concrete. The prestress at the ends of the tendon is zero. The diameter of the tendon regains its original value towards the end over the transmission length. The change of diameter from the original value (at the end) to the reduced value (after the transmission length), creates a wedge effect in concrete. This helps in the transfer of prestress from the tendon to the concrete. This is known as the Hoyer effect. The following figure shows the sequence of the development of Hoyer effect.
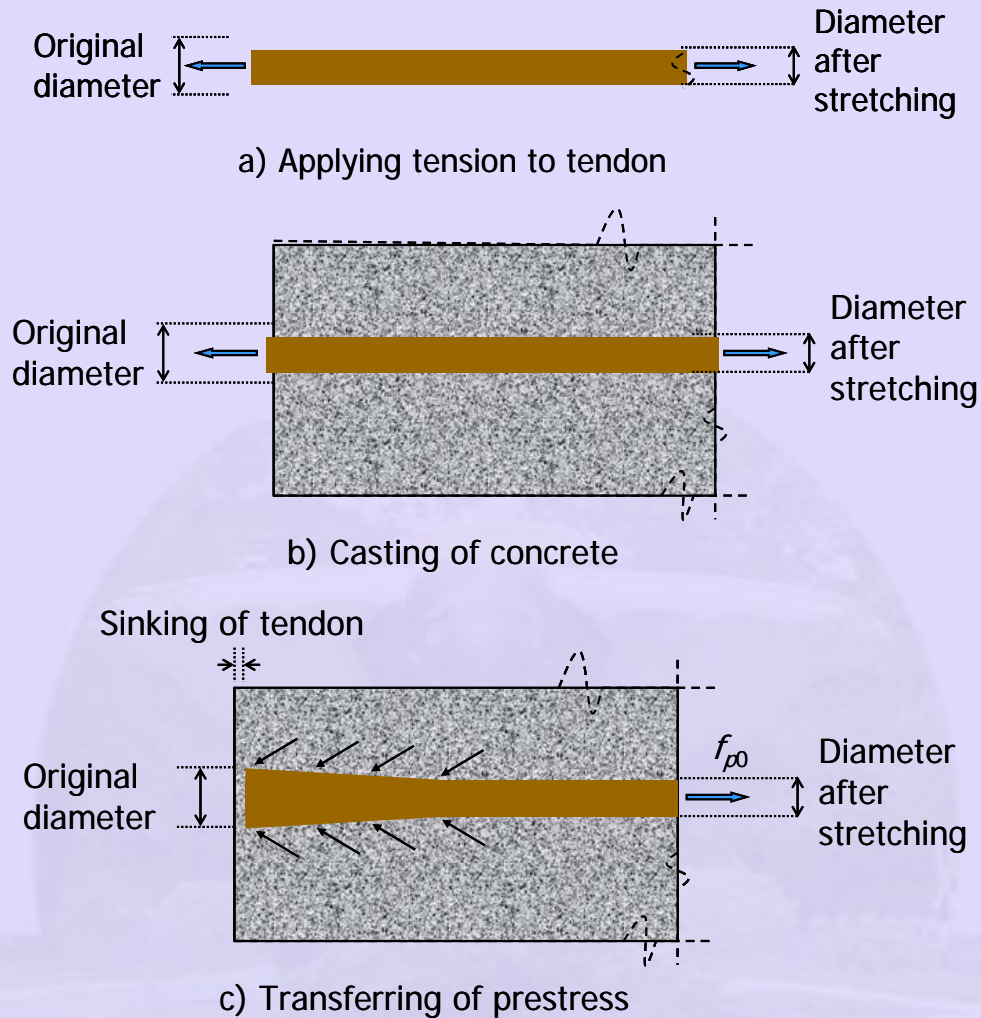
a) Applying tension to tendon

b) Casting of concrete

c) Transferring of prestress

Figure 7-1.4   Hoyer effect

Since there is no anchorage device, the tendon is free of stress at the end. The concrete should be of good quality and adequate compaction for proper transfer of prestress over the transmission length.

Transmission Length

There are several factors that influence the transmission length. These are as follows.

1) Type of tendon
   ➢ wire, strand or bar
2) Size of tendon
3) Stress in tendon
4) Surface deformations of the tendon

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> ➤ Plain, indented, twisted or deformed
5) Strength of concrete at transfer
6) Pace of cutting of tendons
   > ➤ Abrupt flame cutting or slow release of jack
7) Presence of confining reinforcement
8) Effect of creep
9) Compaction of concrete
10) Amount of concrete cover.

The transmission length needs to be calculated to check the adequacy of prestress in the tendon over the length. A section with high moment should be outside the transmission length, so that the tendon attains at least the design effective prestress (f_{pe}) at the section. The shear capacity at the transmission length region has to be based on a reduced effective prestress.

IS:1343 - 1980 recommends values of transmission length in absence of test data. These values are applicable when the concrete is well compacted, its strength is not less than 35 N/mm^2 at transfer and the tendons are released gradually. The recommended values of transmission length are as follows.

Table 7-1.1   Values of transmission length

For plain and intended wires	$L_t = 100 \; \Phi$
For crimped wire	$L_t = 65 \; \Phi$
For strands	$L_t = 30 \; \Phi$

Here, Φ is the nominal diameter of the wire or strand.

To avoid the transmission length in the clear span of a beam, **IS:1343 - 1980** recommends the following.

1) To have an overhang of a simply supported member beyond the support by a distance of at least ½ L_t.

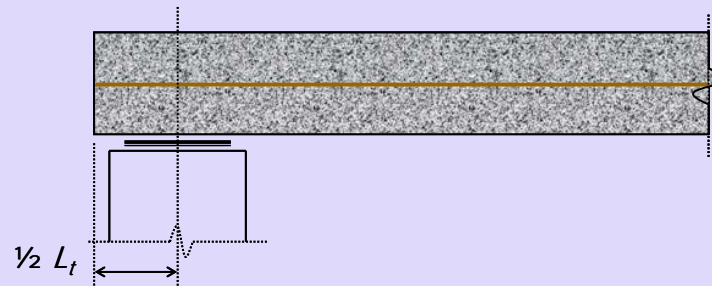5   www.jntufastupdates.com

Figure 7-1.5   End of a simply supported member

2) If the ends have fixity, then the length of fixity should be at least L_t.
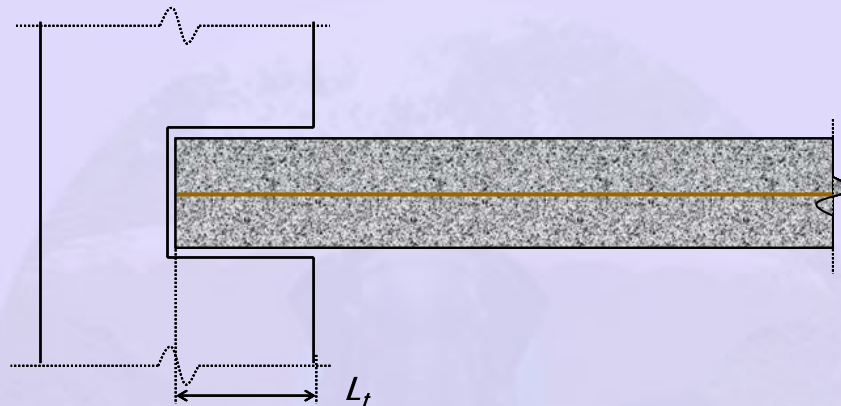

Figure 7-1.6   End of a member with fixity

Development Length

The development length needs to be provided at the critical section, the location of maximum moment. The length is required to develop the ultimate flexural strength of the member.   The development length is the minimum length over which the stress in tendon can increase from zero to the ultimate prestress (f_{pu}). The development length is significant to achieve ultimate capacity.

If the bonding of one or more strands does not extend to the end of the member (de-bonded strand), the sections for checking development of ultimate strength may not be limited to the location of maximum moment.

The development length (L_d) is the sum of the transmission length (L_t) and the bond length (L_b).

$$L_d = L_t + L_b \qquad (7\text{-}1.1)$$

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The bond length is the minimum length over which, the stress in the tendon can increase from the effective prestress (f_{pe}) to the ultimate prestress (f_{pu}) at the critical location.

The following figure shows the variation of prestress in the tendon over the length of a simply supported beam at ultimate capacity.
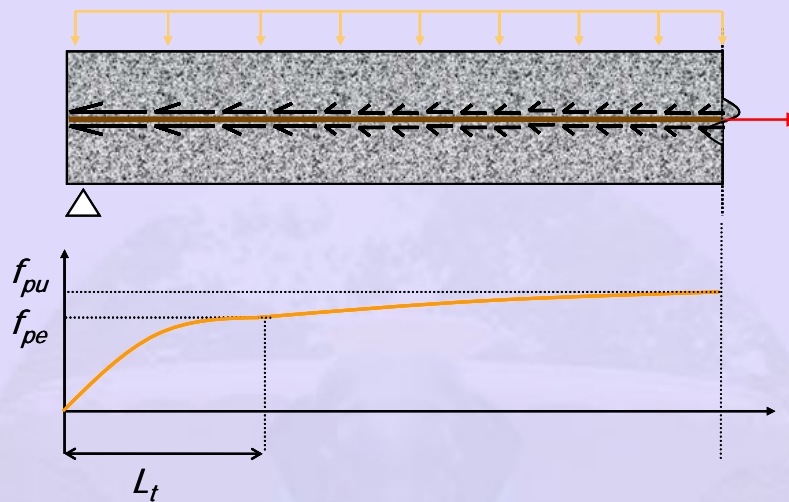

Figure 7-1.7   Variation of prestress in tendon at ultimate

The calculation of the bond length is based on an average design bond stress (τ_{bd}). A linear variation of the prestress in the tendon along the bond length is assumed. The following sketch shows a free body diagram of a tendon along the bond length.
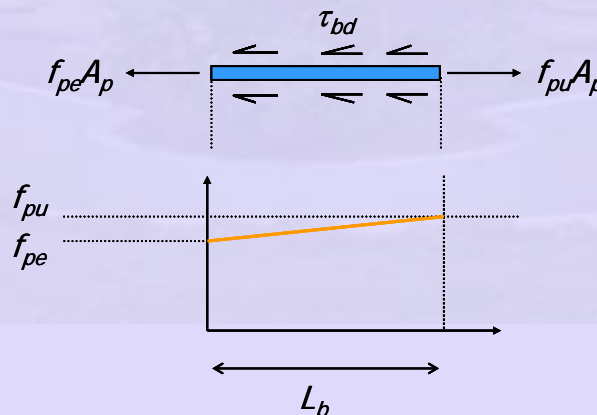

Figure 7-1.8   Assumed variation of prestress in tendon along the bond length

The bond length depends on the following factors.

    1) Surface condition of the tendon

    2) Size of tendon

    3) Stress in tendon

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4) Depth of concrete below tendon

From equilibrium of the forces in the above figure, the expression of the bond length is derived.

$$L_b = \frac{\left(f_{pu} - f_{pe}\right)\varphi}{4\tau_{bd}}$$

(7-1.2)

Here, Φ is the nominal diameter of the tendon.

The value of the design bond stress (τ_{bd}) can be obtained from **IS:456 - 2000, Clause 26.2.1.1**. The table is reproduced below.

Table 7-1.2   Design bond stress for plain bars

Grade of concrete	M30	M35	M40 and above
τ_{bd} (N/mm^2)	1.5	1.7	1.9

End Zone Reinforcement

The prestress and the Hoyer effect cause transverse tensile stress (σ_t). This is largest during the transfer of prestress. The following sketch shows the theoretical variation of σ_t.
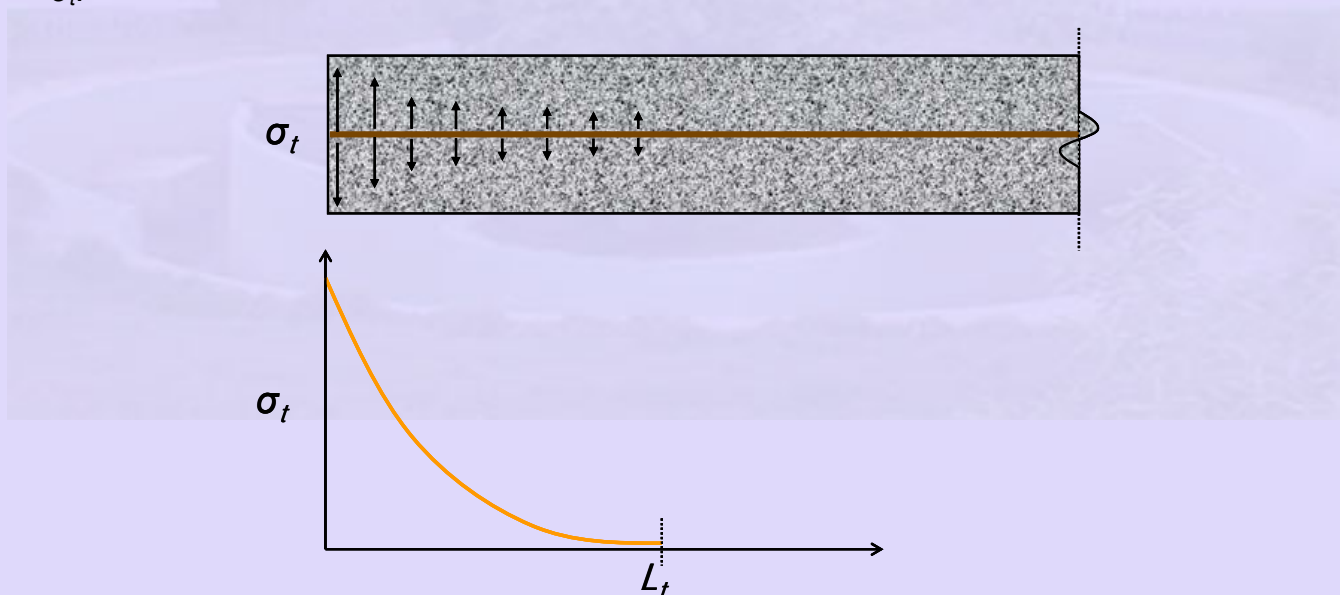

Figure 7-1.9   Transverse stress in the end zone of a pre-tensioned beam

To restrict the splitting of concrete, transverse reinforcement (in addition to the reinforcement for shear) needs to be provided at each end of a member along the

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transmission length. This reinforcement is known as **end zone reinforcement**.

The generation of the transverse tensile stress can be explained by the free body diagram of the following zone below crack, for a beam with an eccentric tendon. Tension (*T*), compression (*C*) and shear (*V*) are generated due to the moment acting on the horizontal plane at the level of the crack. The internal forces along the horizontal plane are shown in (a) of the following figure. The variation of moment (due to the couple of the normal forces) at horizontal plane along the depth is shown in (b).
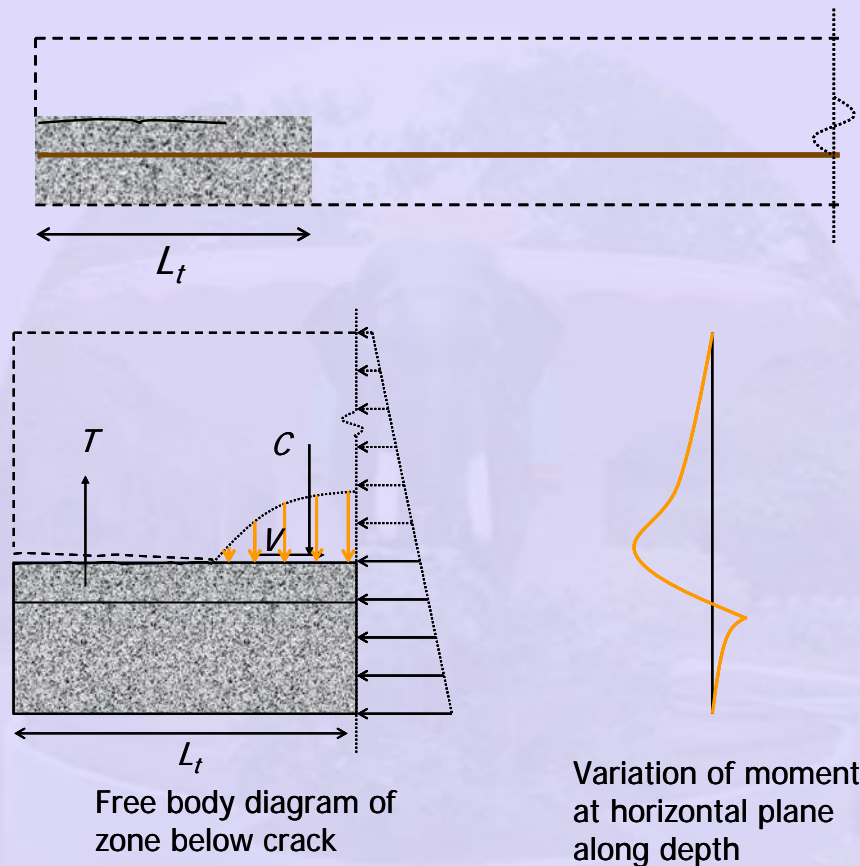

Figure 7-1.10   Forces in the end zone

The end zone reinforcement is provided to carry the tension (*T*) which is generated due to the moment (*M*). The value of *M* is calculated for the horizontal plane at the level of CGC due to the compressive stress block from the normal stresses in a vertical plane above CGC. The minimum amount of end zone reinforcement (A_{st}) is given in terms of the moment (*M*) as follows.

$$A_{st} = \frac{2.5M}{f_s h}$$

<div align="right">(7-1.3)</div>

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In the previous equation,

h = total depth of the section

M = moment at the horizontal plane at the level of CGC due to the compressive stress block above CGC

f_s = allowable stress in end zone reinforcement.

The lever arm for the internal moment is h/2.5. The value of f_s is selected based on a maximum strain.

The end zone reinforcement should be provided in the form of closed stirrups enclosing all the tendons, to confine the concrete. The first stirrup should be placed as close as possible to the end face, satisfying the cover requirements. About half the reinforcement can be provided within a length equal to $\frac{1}{3}L_t$ from the end. The rest of the reinforcement can be distributed in the remaining $\frac{2}{3}L_t$.

References:

1) Krishnamurthy, D. "*A Method of Determining the Tensile Stresses in the End Zones of Pre-tensioned Beams*", Indian Concrete Journal, Vol. 45, No. 7, July 1971, pp. 286-297.

2) Krishnamurthy, D. "*Design of End Zone Reinforcement to Control Horizontal Cracking in Pre-tensioned Concrete Members at Transfer*", Indian Concrete Journal, Vol. 47, No. 9, September 1973, pp. 346-349.

Example 7-1.1

Design the end zone reinforcement for the pre-tensioned beam shown in the following figure.

The sectional properties of the beam are as follows.

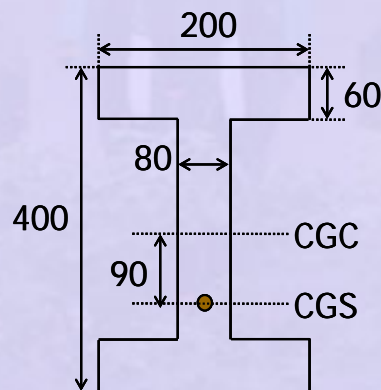A = 46,400 mm^2

I = 8.47 × 108 mm^4

Z = 4.23 × 105 mm^3

There are 8 prestressing wires of 5 mm diameter.

A_p = 8 × 19.6 = 157 mm^2

The initial prestressing is as follows.

f_{p0} = 1280 N/mm^2.

Limit the stress in end zone reinforcement (f_s) to 140 N/mm^2.

Cross-section at end

Solution

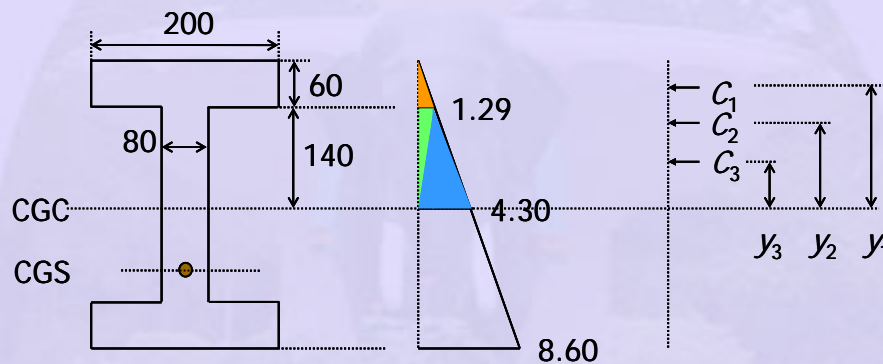1) Determination of stress block above CGC

Initial prestressing force

$$P_0 = A_p.f_{po}$$
$$= 157 \times 1280 \text{ N}$$
$$= 201 \text{ kN}$$

Stress in concrete at top

$$f_t = -\frac{P_0}{A} + \frac{P_0 e}{Z}$$

$$= -\frac{201 \times 10^3}{46400} + \frac{201 \times 10^3 \times 90}{4.23 \times 10^5}$$

$$\approx 0 \, N/mm^2$$

Stress at bottom

$$f_b = -\frac{P_0}{A} - \frac{P_0 e}{Z}$$

$$= -\frac{201 \times 10^3}{46400} - \frac{201 \times 10^3 \times 90}{4.23 \times 10^5}$$

$$= -8.60 \, N/mm^2$$

Stress profile   Components of
                          compression
                          block

2) Determination of components of compression block

C_1 $= \frac{1}{2} \times 1.29 \times 200 \times 60$

$= 7.74 \, kN$

y_1 $= 140 + \frac{1}{3} \times 60$

$= 160 \, mm$

C_2 $= \frac{1}{2} \times 1.29 \times 140 \times 80$

$= 7.22 \, kN$

y_2 $= \frac{2}{3} \times 140$

$= 93.3 \, mm$

C_3 $= \frac{1}{2} \times 4.3 \times 140 \times 80$

$= 24.08 \, kN$

y_3 $= \frac{1}{3} \times 140$

$= 46.7 \, mm$

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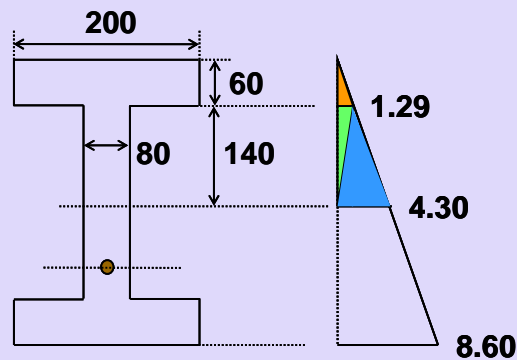
3) Determination of moment

M $= \Sigma\ C_i.y_i$

$= C_1.y_1 + C_2.y_2 + C_3.y_3$

$= (7.74 \times 160) + (7.22 \times 93.3) + (24.08 \times 46.7)$

$= 3036.6$ kN-mm

4) Determination of amount of end zone reinforcement

$$A_{st} = \frac{2.5M}{f_s h}$$

$$= \frac{2.5M}{140 \times 400}$$

$$= \frac{2.5 \times 3036.6 \times 10^3}{140 \times 400}$$

$$= 135.6\ \text{mm}^2$$

With 6 mm diameter bars, required number of 2 legged closed stirrups

$= 135.6\ /\ (2 \times 28.3) \Rightarrow 3.$

For plain wires, transmission length

L_t $= 100\ \Phi$

$= 500$ mm.

Provide 2 stirrups within distance 250 mm ($L_t/2$) from the end. The third stirrup is in the next 250 mm.

Designed end zone reinforcement

(3) 6 mm diameter stirrups

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